PHIL 183 (F13) – TA section
Symbolization and Derivation
Chapter III

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Office Hours: W 1pm-2pm
F 10:50am-11:50am

↪ Stylistic variants:

$\bigwedge x\ Fx$	Everything is F. Each thing is F. All things are F. For each x, x is F.
$\bigvee x\ Fx$	Something is F. At least one thing is F. There is a F.
$\bigvee x\ (Fx \wedge Gx)$	Some F is G. Some F's are G's. At least one F is G. There is a F who/which is G. **A (certain) F is G. **
$\bigvee x\ (Fx \wedge \sim Gx)$	Some F isn't G.
$\bigwedge x\ (Fx \rightarrow Gx)$	All F's are G. Each F is G. Every F is G. Everything that is F is G. Anything that is F is G. A (generic) F is G. **Any F is G. ** If anything is F, it is G. Anyone who is F is G. Whoever/Whatever is F is G. The only F's are G.
$\bigwedge x\ (Gx \rightarrow Fx)$ $\sim\bigvee x\ (\sim Fx \wedge Gx)$	Only F's are G's. None but F's are G's. Nothing other/but F's are G's.
$\bigwedge x(Gx \leftrightarrow Fx)$ $\bigwedge x(Gx \rightarrow Fx) \wedge \bigwedge x(Fx \rightarrow Gx)$	All and only F's are G's.
$\sim\bigvee x\ (Fx \wedge Gx)$	No F is G.
$\bigwedge x\ (Fx \rightarrow \sim Gx)$	F's are not G.

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Symbolization	Sentence
$\bigwedge x\,(Fx \rightarrow Gx) \wedge \bigwedge x\,(Ix \rightarrow Gx)$ $\bigwedge x\,(Fx \vee Ix \rightarrow Gx)$	All F's and I are G.
$\bigwedge x\,(Ix \rightarrow Fx \wedge Gx)$	All I's are F's and G's.
$\bigwedge x\,(Ix \rightarrow {\sim}Fx \vee Gx)$ ${\sim}\bigvee x\,(Ix \wedge Fx \wedge {\sim}Gx)$	No I is F unless he/she/it is G. No I's are F's unless they are G's.
$\bigwedge x\,(Ix \rightarrow ({\sim}Fx \vee GA)$ $\bigwedge x\,(Ix \rightarrow {\sim}Fx) \vee GA$ ${\sim}\bigvee x\,(Ix \wedge Fx \wedge {\sim}GA)$ ${\sim}\bigvee x\,(Ix \wedge Fx) \wedge {\sim}GA$	No I is F unless A is G.
$\bigwedge x\,(Hx \rightarrow (\ Gx \rightarrow Fx \vee Ix))$	Among H's, only F's and I's are G.
$\bigwedge x\,(Ix \wedge Gx \rightarrow Fx)$	I's who are G's are F's.
$\bigwedge x\,(Ix \rightarrow Fx) \wedge \bigwedge x\,(Ix \rightarrow Gx)$ $\bigwedge x\,(Ix \rightarrow Gx \wedge Fx)$	I's, who are G's, are F's.
$\bigwedge x\,(Ix \rightarrow Gx) \rightarrow \bigwedge x\,(Fx \rightarrow Gx)$	If every I is G, then any F is G.
$\bigvee x\,(Fx \wedge Gx) \rightarrow \bigwedge x\,(Ix \rightarrow Gx)$	If only F's are G's, then every I is G.
$GA \rightarrow \bigwedge x\,(Fx \rightarrow Gx)$	If A is G, then any F is G.
$\bigvee x\,(Fx \wedge Gx) \rightarrow GA$	If any F is G, then A is G.
$\bigwedge x\,(Fx \wedge Gx \rightarrow Hx)$ $\bigwedge x\,(Fx \rightarrow (Gx \rightarrow Hx)$	If any F is G, then he/she/it is H.

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↪ New form of derivation: <u>**U**niversal **D**erivation - **UD**</u> (K&M, p. 143)

n. ~~Show~~ ⋀α Φα Assertion **n+1**, UD

n+1. ~~Show~~ Φα

⋮

Then follow the appropriate strategy.
- If Φα is a conditional, do CD;
- If Φα is a negation, ID;
- If Φα is a disjunction, derive the corresponding conditional;
- If Φα is a conjunction, derive each of the conjuncts;
- If Φα is a biconditional, derive both directions of the biconditional.

↪ New Rules of Inference (K&M, p. 141)

- proper substitution (K&M, p. 139): Φβ comes from proper substitution of β for α if Φβ is just like Φα except for having <u>free occurrences</u> of β <u>whenever</u> Φα has <u>free occurrences</u> of α.

<u>UI</u>

n. ⋀α Φα

Φβ n, UI/β

** Where Φβ comes from Φα by proper substitution of the term β for the variable α in Φα **

<u>EG</u>

n. Φβ

⋁α Φα n, EG

** Where Φβ comes from Φα by proper substitution of the term β for the variable α in Φα **

<u>EI</u>

n. ⋁α Φα

Φβ n, EI/β

* Where Φβ comes from Φα by proper substitution of the term β for the variable α in Φα; AND
* β is a variable; AND
* β is a new variable, i.e., doesn't occur anywhere in the derivation.

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- Examples:

** In order to follow a derivation, you have to read carefully the annotation. Make sure you know which lines and inference rules are being used to justify a line **

Deriv 3.001: ⋀x (Fx ∧ Gx) ∴ ⋀xGx

1.	~~Show~~ ⋀xGx	2, UD
2.	~~Show~~ Gx	4, DD
3.	Fx ∧ Gx	Pr UI/x
4.	Gx	3, S

Once I show Gx, I can box and cancel line 1.

Remember: UD is not an inference rule !!

Deriv 3.002: ⋁xFx ∴ ⋁x(Gx⟶Fx)

1.	~~Show~~ ⋁x(Gx⟶Fx)	4 DD
2.	Fa	Pr EI/a
3.	Ga⟶Fa	2 RT2
4.	⋁x(Gx⟶Fx)	3 EG

Hint: EI as soon as you can !!

Deriv 3.004: ⋀x(Fx⟶Gx). ⋀x(Gx⟶Hx) ∴ FA⟶⋁x(Gx ∧ Hx)

1.	~~Show~~ FA⟶⋁x(Gx ∧ Hx)	3, CD
2.	FA	Ass CD
3.	~~Show~~ ⋁x(Gx ∧ Hx)	10, 11 ID
4.	~⋁x(Gx ∧ Hx)	Ass ID
5.	⋀x ~(Gx ∧ Hx)	4, QN
6.	FA⟶GA	Pr1, UI/A
7.	GA	2, 6 MP
8.	GA⟶HA	Pr2, UI/A
9.	HA	6, 7 MP
10.	~(GA ∧ HA)	5 UI/A
11.	GA ∧ HA	7,9 ADJ

Deriv 3.014: ⋀x(Fx⟶Gx). ⋁x((Fx ∧ Hx) ∨ (Fx ∧ Jx))⟶~⋀x(Fx⟶Gx) ∴ ⋀x(Fx⟶~Jx)

1.	~~Show~~ ⋀x(Fx⟶~Jx)	2 UD
2.	~~Show~~ Fx⟶~Jx	4 CD
3.	Fx	Ass CD
4.	~~Show~~ ~Jx	10, 11 ID
5.	Jx	Ass ID
6.	~⋁x((Fx ∧ Hx) ∨ (Fx ∧ Jx))	Pr1 DN, Pr2 MT
7.	⋀x~((Fx ∧ Hx) ∨ (Fx ∧ Jx))	6 QN
8.	~((Fx ∧ Hx) ∨ (Fx ∧ Jx))	7 UI/x
9.	~(Fx ∧ Hx) ∧ ~(Fx ∧ Jx)	8 DM
10.	~(Fx ∧ Jx)	9 S
11.	Fx ∧ Jx	3,5 ADJ

Remember: to use UD, you have to show the formula that follows the quantifier, in this case, Fx⟶~Jx. For this reason I introduced a new show line. (line 2)

Just our usual strategy to show a negation, i.e., assume the thing that is being negated. (line 5)

Remember: no restrictions upon the term you instantiate by UI ‼ (line 8)

Deriv 3.715: ⋀x⋁y(Fx ∨ ~Gy). ⋁x⋀y(Gy ∨ Hx) ∴ ~⋁xHx⟶⋀xFx

1.	~~Show~~ ~⋁xHx⟶⋀xFx	3 CD
2.	~⋁xHx	Ass CD
3.	~~Show~~ ⋀xFx	4 UD
4.	~~Show~~ Fx	10, 12 Id
5.	~Fx	Ass ID
6.	Fx ∨ ~Ga	Pr1 UI/x, EI/a
7.	~Ga	5, 6 MTP
8.	⋀y(Gy ∨ Hb)	Pr2 EI/b
9.	Ga ∨ Hb	8 UI/a
10.	Hb	7, 9 MTP
11.	⋀x~Hx	2 QN
12.	~Hb	11 UI/b

Just our usual strategy to show an atomic sentence, i.e., assume the negation. (line 5)

Pr1 UI/x: ⋁y(Fx ∨ ~Gy), then EI/a. (line 6)

Remember the restriction: NEW VARIABLE ‼ (line 8)

I decided to UI to *a* because I have ~Ga on line 7. (line 9)

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↪ Exercises to practice:

* For each of the following expression, state whether or not it is a well formed formula. If an expression is a symbolic formula, give the tree of formation. (Examples: K&M, p.121)

Pars 3.002: $\bigvee x\sim(Fx)$

Pars 3.011: $\bigvee x(E \longrightarrow Fx)$

Pars 3.012: $\bigwedge A(FA \longrightarrow \sim GA)$

Pars 3.017: $\bigwedge a(Hx \longleftrightarrow Gy)$

Pars 3.026: $\sim\bigwedge x\sim\bigvee yFx \wedge \sim Gy$

Pars 3.027: $\bigwedge x(FGx \longrightarrow Gy)$

Pars 3.028: $\bigvee xFx \wedge \bigvee xGx \longrightarrow \bigvee x(Fx \wedge Gx)$

Pars 3.030: $\bigvee x(P \longrightarrow \bigwedge x\sim Qx)$

* Determine which inference rule, if any, the following arguments instantiate:

Recog 3.001: $$\frac{\bigwedge x(Fx \longrightarrow Gy)}{Fx \longrightarrow Gy}$$

Recog 3.002: $$\frac{Gx}{\bigwedge xGx}$$

Recog 3.004: $$\frac{\bigvee yGy}{GA}$$

Recog 3.006: $$\frac{\bigvee xGy}{Gz}$$

Recog 3.007: $$\frac{\bigwedge x\bigvee y(Fx \longrightarrow Gy \vee Hx)}{\bigvee y(FA \longrightarrow Gy \vee HA)}$$

Recog 3.011: $$\frac{FA \longrightarrow GA}{\bigvee y(Fy \longrightarrow Gy)}$$

Recog 3.018: $$\frac{(\bigwedge xFx \longrightarrow \bigvee y(Hy \vee Hx))}{FB \longrightarrow \bigvee y(Hy \vee HB)}$$

Recog 3.020: $$\frac{\bigvee z(FA \wedge Gz) \longrightarrow \bigvee xHx \vee GA}{\bigvee x(\bigvee z(FA \wedge Gz) \longrightarrow \bigvee xHx \vee Gx)}$$

Recog 3.028: $$\frac{\bigvee xFx \longrightarrow FA \vee \bigwedge xGx}{\bigvee x(\bigvee xFx \longrightarrow Fx \vee \bigwedge xGx)}$$

Recog 3.030: $$\frac{\bigwedge x(Fx \longrightarrow \bigvee y(FB \wedge Gy))}{\bigwedge x(Fx \longrightarrow \bigvee z\bigvee y(Fz \wedge Gy))}$$

↪ Do as many derivations as you can on the software ‼